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The linear stability in systems with intensive mass transfer—II. Gas–liquid

CHR. BOYADJIEV and I. HALATCHEV
Bulgarian Academy of Science, Institute of Chemical Engineering, Sofia 1113, Bl. 103, Bulgaria

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Abstract—A linear analysis of the stability of the flow in a laminar boundary layer under conditions of intensive interphase mass transfer, when high mass fluxes through the gas–liquid phase boundary induce secondary flows, is suggested. Essential interaction between flows in gas and liquid is considered in the case of a movable liquid surface. The critical Reynolds numbers are obtained at different intensities of non-linear mass transfer in a laminar boundary layer in the gas. The influence of the direction of the intensive interphase mass transfer on the hydrodynamic stability is analogous to the one in the case of motionless interface. The motion of the interface leads to a decrease of velocity gradients, which is the cause for increase in stability of the gas flow. Flow is stable at large Reynolds number in the liquid phase. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

The sufficient changes in the hydrodynamic stability of a flow in the boundary layer along a solid surface due to the induction of secondary flows on the gas–solid boundary, as a result of an intensive interphase mass transfer between two phases, was discussed in the first report [1]. Essential interaction between flows in gas and liquid will be observed if a movable liquid surface replaces the unmovable solid surface. There will also be the effect of induction of secondary flows as a result of intensive interphase mass transfer in the gas–liquid systems, but this effect is superposed with the hydrodynamic interaction between the above mentioned two phases. The stability under these conditions is not only of theoretical, but also of practical, interest before the fact that it defines the rate of the number of industrial processes of absorption and desorption.

2. NON-LINEAR MASS TRANSFER

The mathematical model of the non-linear mass transfer in gas–liquid systems will be considered in the approximation of the boundary layer theory [2, 3], taking into account that the diffusive resistance is in the gas phase [4]. It was shown that the non-linear effects in the liquid can be omitted in ref. [5]. Hence, we will focus our attention on the problem illustrated in Fig. 1. The mathematical description has the following form:

$$u_j\frac{\partial u_j}{\partial x}+v_j\frac{\partial u_j}{\partial y}=\nu_j\frac{\partial^2 u_j}{\partial y^2},$$

$$\frac{\partial u_j}{\partial x}+\frac{\partial v_j}{\partial y}=0,\quad j=1,2$$

$$u_1\frac{\partial c}{\partial x}+v_1\frac{\partial c}{\partial y}=D_1\frac{\partial^2 c}{\partial y^2};\tag{1}$$

where u is velocity in the x direction, v is velocity in the y direction, ν is viscosity, D is the diffusion coefficient and the indexes 1, 2 denote the gas and the liquid, respectively.

The boundary conditions of (1) express the continuity of the velocity and the stress tensor on the face boundary, where a thermodynamic equilibrium is established

$$x=0,\quad u=u_{j0},\quad c=c_0,\quad j=1,2;$$

$$y=0,\ u_1=u_2,\ c=c^*,\ \mu_1\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y},$$

$$v_1=-\frac{MD_1}{\rho^*_{0j}}\frac{\partial c}{\partial y},\quad v_2=0;$$

$$y\to\infty,\quad u_1=u_{10},\quad c=c_0,$$

$$y\to-\infty,\quad u_2=u_{20},\tag{2}$$

where μ is viscosity, c is the concentration of the transferred substance, M is the molecular mass, and the indexes 0 and * denote the values in volume and at the interface, respectively.

The problem (1) and (2) was solved numerically and asymptotically [4–6].

$$u_j=0.5ju_{j0}\varepsilon_j\Phi'_j,$$

$$v_j=(-1)^{j-1}0.5j\left(\frac{u_{j0}\nu_j}{x}\right)^{0.5}(\xi_j\Phi'_j-\Phi_j),$$

$$c=c_0-(c_0-c^*)\Psi,$$

$$\Phi_j=\Phi_j(\eta_j),\quad \Psi_j=\Psi_j(\eta_j),$$

NOMENCLATURE

a	initial value of the Blasius function
A	dimensionless wave number
b	initial value of first derivative of the Blasius function
c	concentration, second derivative of the Blasius function
C	dimensionless phase velocity
D	diffusion coefficient
f	Blasius function
i	imaginary number
k	parameter
M	molecular mass
Re	Reynolds number
Sc	Schmidt number
u	velocity of basic stationary flow in x direction
v	velocity of basic stationary flow in y direction
x	coordinate
y	coordinate.

Greek symbols

γ	parameter
ε	parameter
η	similarity variable
θ	parameter
ν	kinematic viscosity
ξ	variable
ρ	density
φ	dimensionless amplitude
Φ	stream function in x direction
Ψ	stream function in y direction.

Subscripts and superscripts

*	conditions on interface
0	conditions in volume
1	gas phase
2	liquid phase
cr	critical number
i	imaginary part of complex number
max	maximum
min	minimum
r	real part of complex number.

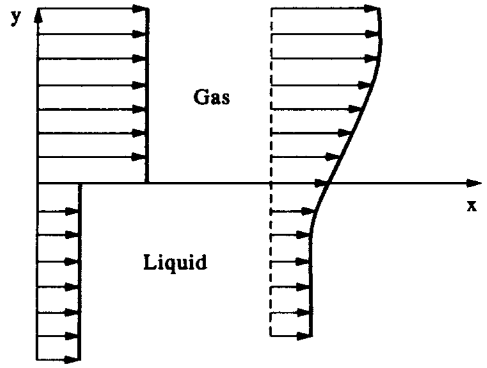

Fig. 1. Velocity profiles in the gas and liquid flows in the boundary layer (gas–liquid system).

$$\eta_j = (-1)^{j-1} y \left(\frac{u_{j0}}{4D_j x}\right)^{0.5},$$

$$\varepsilon_j = Sc_j^{0.5}, \quad Sc_j = \frac{\nu_j}{D_j}, \quad j = 1,2. \tag{3}$$

Substitution of equation (3) into equations (1) and (2) leads to

$$\Phi_1''' + \varepsilon_1^{-1}\Phi_1\Phi_1'' = 0, \quad \Phi_2''' + 2\varepsilon_2^{-1}\Phi_2\Phi_2'' = 0,$$

$$\Psi'' + \varepsilon_1\Phi_1\Psi' = 0, \Phi_1(0) = -\theta_3\Psi'(0), \quad \Phi_2(0) = 0,$$

$$\Phi_1'(\infty) = \frac{2}{\varepsilon_1}, \quad \Phi_2'(\infty) = \frac{1}{\varepsilon_2}, \Phi_1'(0) = 2\theta_1\frac{\varepsilon_2}{\varepsilon_1}\Phi_2'(0),$$

$$\Phi_2''(0) = -0.5\theta_2\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2\Phi_1''(0),$$

$$\Psi(0) = 1, \quad \Psi(\infty) = 0, \tag{4}$$

where

$$\theta_1 = \frac{u_{20}}{u_{10}}, \quad \theta_2 = (\mu_1/\mu_2)(\nu_1/\nu_2)^{-0.5}(u_{10}/u_{20})^{1.5}$$

$$\theta_3 = \frac{M(c_0 - c^*)}{\varepsilon_1\rho_0^*}. \tag{5}$$

The solution of equation (4) is obtained [4, 5] by determination of the initial values of f, which allows us to further define the velocity profiles in the gas and liquid, as solutions of a problem with initial conditions:

$$2f_j''' + f_j f_j'' = 0, \quad f_j = f_j(\xi_j), \quad \xi_j = \frac{2}{\varepsilon_j}\eta_j,$$

$$f_j(0) = a_j, \quad f_j'(0) = b_j,$$

$$f_j''(0) = c_j, \quad (f_j'(\infty) = 1), \quad j = 1,2 \tag{6}$$

where

$$a_1 = a_{10}, \quad b_1 = \frac{\varepsilon_1}{2}b_{10}, \quad c_1 = \frac{\varepsilon_1^2}{4}c_{10},$$

$$a_2 = 0, \quad b_2 = \varepsilon_2 b_{20}, \quad c_2 = -\frac{\varepsilon_2^2}{2}c_{20}, \tag{7}$$

Table 1. Initial values of f, its derivatives and parameter k of the gas flow ($\varepsilon_1 = 1, \theta_1 = 0.1, \theta_2 = 0.152, \theta_3 = \theta$)

θ	a_{10}	b_{10}	c_{10}	k
−0.3	0.2797	0.2185	1.662	0.953
−0.2	0.1703	0.2166	1.520	1.133
−0.1	0.07852	0.2152	1.402	1.301
0	0	0.2138	1.304	1.428
0.1	−0.06822	0.2129	1.220	1.552
0.2	−0.1283	0.2118	1.084	1.665
0.3	−0.1816	0.2107	1.084	1.768

Table 2. Initial values of f, its derivatives and parameter k of the liquid flow ($\varepsilon_2 = 20, \theta_1 = 0.1, \theta_2 = 0.152, \theta_3 = \theta$)

θ	b_{20}	c_{20}	k
−0.3	0.0546	0.00033	−0.1
0	0.0536	0.00026	−0.086
0.3	0.0527	0.00022	−0.13

while the values of a_{10}, b_{10}, c_{10}, b_{20} and c_{20} for $\varepsilon_1 = 1$, $\varepsilon_2 = 20$, $\theta_1 = 0.1$, $\theta_2 = 0.152$ are given in Tables 1 and 2.

3. STABILITY ANALYSIS

In the present paper the influence of non-linear mass transfer in the gas phase on the stability of the flows in the gas–liquid system will be discussed.

It was shown in ref. [1] that the Orr–Sommerfeld equation in the approximation of almost parallel flows has the same form for the gas and the liquid, as follows:

$$(f'-C)(\varphi''-A^2\varphi)-f'''\varphi$$

$$= -\frac{i}{A\,Re}\left\{(\varphi^{\prime v}-2A^2\varphi''+A^4\varphi)-\tfrac{1}{2}(\xi f'-f)\varphi'''\right.$$

$$\left.+\left[\frac{1}{2}(\xi f'''+f'')+\frac{A^2}{2}(\xi f'-f)\right]\varphi'\right\}, \quad (8)$$

$$\xi = 0, \quad \varphi = 0, \quad \varphi' = 0;$$

$$\xi = \xi_\infty \geqslant 6, \quad (\varphi''-A^2\varphi)'-\gamma(\varphi''-A^2\varphi) = 0,$$

$$(\varphi''-\gamma^2\varphi)'+A(\varphi''-\gamma^2\varphi) = 0,$$

$$\gamma = \frac{k}{4}-\frac{\sqrt{k^2+16A[A+i\,Re(1-C)]}}{4},$$

$$k = \lim_{\xi\to\infty}(\xi f'-f),$$

where

$$f(\xi) = f_j(\xi_j), \quad \xi = \xi_j, \quad \varphi = \varphi_j,$$

$$\gamma = \gamma_j, \quad k = k_j, \quad j = 1, 2. \quad (9)$$

Values of k_j ($j = 1, 2$) are calculated and shown in Tables 1 and 2.

4. RESULTS AND DISCUSSION

The neutral curves (Re, A) and (Re, C) for the gas are shown in Figs. 2 and 3. Critical Reynolds numbers, corresponding wave numbers and phase velocities are presented in Table 3.

It is clearly seen, that the direction of intensive interphase mass transfer influences the hydrodynamic stability of the flow in the boundary layer in the gas

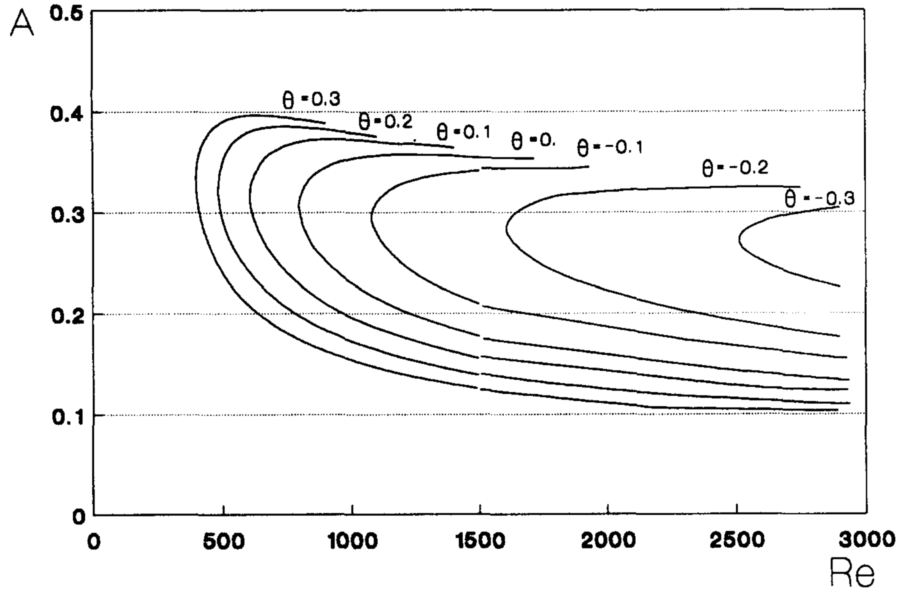

Fig. 2. The neutral curves for the wave number A as a function of the Reynolds number Re (in the gas phase).

Table 3. Values of the critical Reynolds numbers Re_{cr}, wave velocities C_r, wave numbers A and $C_{r\,min}$, A_{min} obtained (in the gas phase)

θ	Re_{cr}	A	C_r	A_{min}	$C_{r\,min}$
−0.3	2511	0.270	0.3863	0.304	0.3878
−0.2	1605	0.285	0.4095	0.325	0.4108
−0.0	1078	0.295	0.4264	0.341	0.4281
0	795	0.305	0.4469	0.356	0.4493
0.3	397	0.330	0.4866	0.398	0.4902
0.2	483	0.320	0.4749	0.386	0.4786
0.1	605	0.315	0.4620	0.373	0.4645

phase analogously to the case [1] at the conditions of a motionless face boundary. Hence, in the case of absorption ($\theta_3 > 0$) the rise of stability is observed. In the opposite case of desorption ($\theta_3 < 0$)—the stability decreases. The motion of interface ($f'(0) > 0$) leads to a decrease of velocity gradients, which is the cause for stabilization of the flow in all cases (increase of Re_{cr}).

The solution of equation (8) for the liquid phase ($f = f_2$) shows that the flow is stable at the large Reynolds numbers ($Re \approx 25000$), which can be explained by the fact that the velocity gradient in the liquid

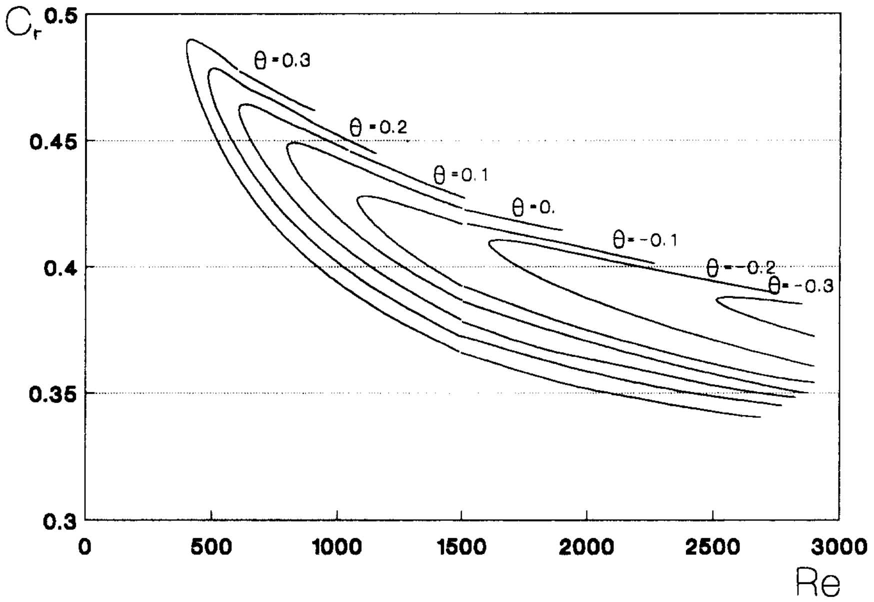

Fig. 3. The neutral curves for the phase velocity C_r as a function of the Reynolds number Re (in the gas phase).

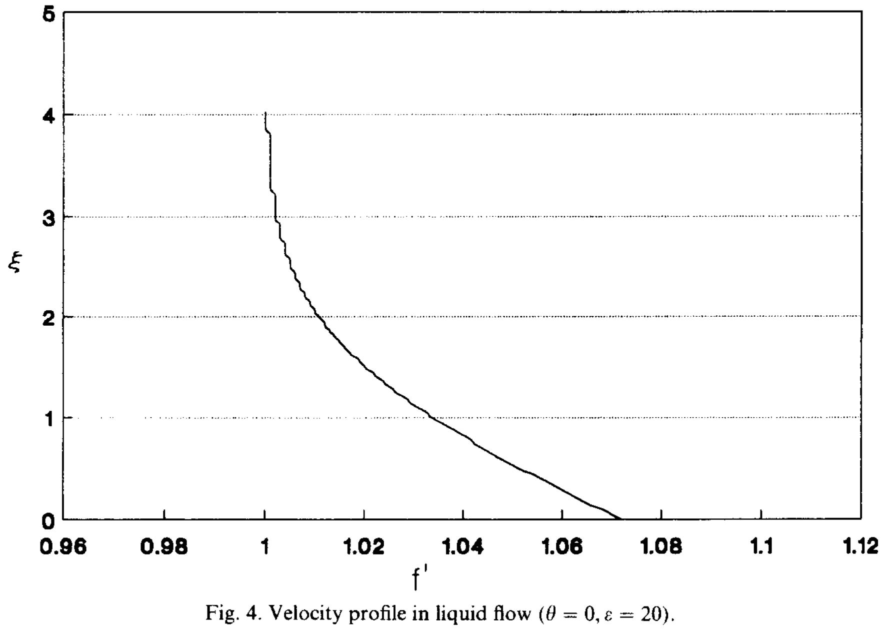

Fig. 4. Velocity profile in liquid flow ($\theta = 0, \varepsilon = 20$).

boundary layer is low (Fig. 4) and shaped as the profile of the Couette flow.

The effects of the intensive interphase mass transfer in gas–liquid systems show themselves as a difference in rates of absorption and desorption. In the cases where the process is limited by the diffusion resistance in the gas phase, this difference is explained by the Marangoni effect, which manifests itself in the liquid phase. The higher rate of absorption (compared with the adsorption) can be made clear by the effect of non-linear mass transfer, i.e. the influence of induced secondary flow on the kinetics of mass transfer. Cases where the desorption rate is higher than the absorption rate can be explained by loss of stability and transition to turbulence, since the flow in the gas phase is able to be turbulent for desorption and laminar for desorption at equal Reynolds numbers.

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